

Important Algebra Skills

1. Using a formula (substitution)

How to use a formula

What is the 7th triangular number? First, write the formula.

$$T = \frac{n(n+1)}{2}$$

Then replace letters with numbers.

$$T = \frac{7(7+1)}{2}$$

$$T = \frac{7 \times 8}{2} = 28$$

Finally, answer the question.

The 7th triangular number is 28.

Calculate

1. $S = \frac{R}{2}(3R - 1)$ where R is the number of rows and S is the number of squares in a design.

Calculate S for an 11-row design.

2. $F = \frac{n(n+1)}{2}$ where F = flowers and n = number of edges.

Calculate F for 6 edges.

3. Einstein said $E = mc^2$ where E is the total energy in joules contained within an object, m is the object's mass in kg and $c \approx 2.99 \times 10^8 \text{ ms}^{-1}$. Find the total energy in a 0.3 kg hamburger.

4. The number of handshakes, N , made between p people is $N = \frac{p(p-1)}{2}$.

Evaluate N for a class of 30 people.

5. The volume of sphere is given by the formula $V = \frac{4}{3} \cdot \pi \cdot r^3$ where r is the radius.

What volume of ink is needed to fill a paintball with a radius of 2 cm?

6. Pythagoras said that $h = \sqrt{a^2 + b^2}$ where h is the length of the longest side on a right-angled triangle and a and b are the lengths of the two other shorter sides.

Calculate h if the two short sides in a right-angled triangle are 3.5 m and 4.5 m.

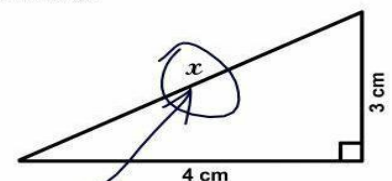
7. The surface area of a sphere is given by $A = 4 \cdot \pi \cdot r^2$ where r is the radius of the sphere.

What is the surface area of the Earth if we assume it is a sphere with radius of 6371 km?

8. The time period in seconds, T , a pendulum takes for one swing is $T = 2\pi \sqrt{\frac{L}{g}}$ where L is the length of the pendulum in metres and $g = 9.80665 \text{ ms}^{-2}$.

Calculate the time period, T , for a 2m long pendulum.

3. Find x .



Here it is

Important Algebra Skills

2. Linear equations with brackets

How to solve linear equations with the variable in brackets

Solve $2(y - 4) = 9$

First expand $2y - 8 = 9$

Opposite of -8 $2y = 9 + 8$

$$2y = 17$$

Opposite of $\times 2$ $y = \frac{17}{2} = 8\frac{1}{2}$ (or 8.5)

Solve

1. $2(x - 3) = 8$

2. $4(t + 3) = 3$

3. $2(x - 5) = -7$

4. $5(y - 4) = 21$

5. $5(t + 2) = 9$

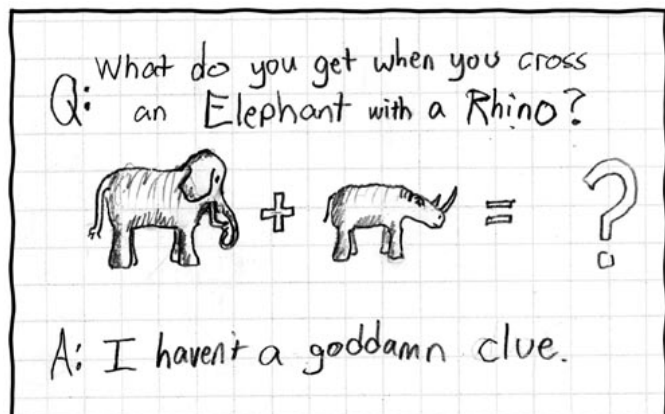
6. $3(n + 2) = -9$

7. $-3(n + 2) = 12$

8. $-5(n - 2) = -12$

9. $4 + 2(x - 3) = 8$

10. $12 - 5(x + 3) = 3$



Important Algebra Skills

3. Linear equations with fractions

How to solve linear equations with fractions

$$\text{Solve } \frac{7x}{2} - 5 = 3$$

First move numbers not in the fraction to the right. (Opposite of -5 .)

$$\frac{7x}{2} = 3 + 5$$

Remove fractions by multiplying all other terms by the denominator.

$$7x = 8 \times 2$$

Next use the opposite of $\times 7$.

$$x = \frac{16}{7} = 2\frac{2}{7} \quad (2.286 \text{ (3dp)})$$

Solve

1. $\frac{5x}{2} - 5 = 3$

2. $\frac{w}{3} - 4 = 5$

3. $\frac{4x+1}{5} = 6$

4. $\frac{3x+2}{5} = 6$

5. $\frac{5x}{2} + 5 = -3$

6. $\frac{m}{5} - 4 = 3$

7. $\frac{5x+2}{3} = 7$

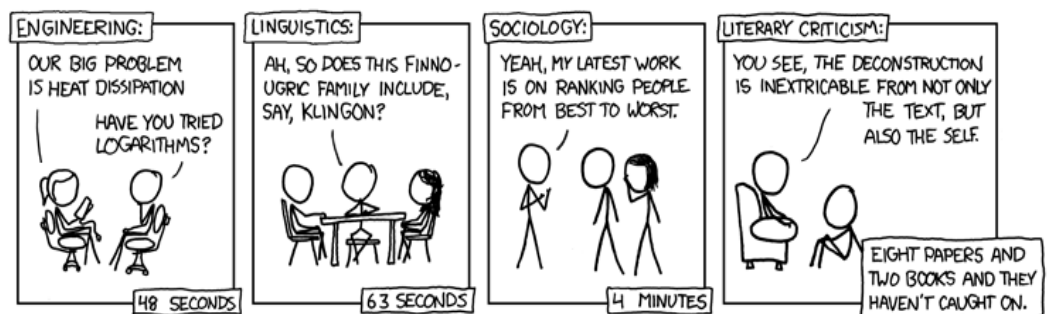
8. $\frac{-4x}{7} - 3 = 10$

9. $\frac{x}{5} - 2 = 0.4$

10. $\frac{w}{3} - 4 = \frac{3}{5}$

MY HOBBY:

SITTING DOWN WITH GRAD STUDENTS AND TIMING HOW LONG IT TAKES THEM TO FIGURE OUT THAT I'M NOT ACTUALLY AN EXPERT IN THEIR FIELD.



Important Algebra Skills 4. Linear equations collecting like terms

How to solve linear equations with like terms to collect

Solve $11x - 7 = 8x + 5$

Use opposites, + and −,
to collect the letters and numbers
on the same side of the = sign.

$$11x - 8x = +5 + 7$$

Combine like terms
(Add or subtract, powers do not change.)

$$3x = 12$$

Use the opposite of X 3

$$x = \frac{12}{3} = 4$$

Solve

1. $13x - 5 = 8x + 3$

2. $5x + 7 = x - 2$

3. $5m - 7 = 2m + 5$

4. $3m + 5 = 3 - 5m$

5. $10x - 7 = 8x + 2$

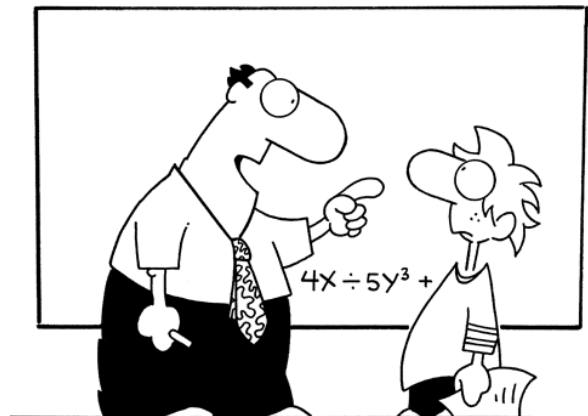
6. $5x + 9 = x - 3$

7. $3m - 9 = 2m + 1$

8. $6n - 2 = 3 + 3n$

9. $5m - 7 = 2m - 0.4$

10. $5 - 5a + 1 = 3 - 10a + 2a$



"Algebra will be important to you later in life.
When you're my age, you can use it to
torture people your age!"

Important Algebra Skills

5. Multiplying and expanding

How to multiply algebraic expressions

Numbers X numbers, letters X letters, add powers. $3x^2y \cdot 4x^3 = 12x^5y$

Each term inside a bracket gets multiplied. $-3(x+2) = -3x-6$

Multiply (expand) brackets before collecting terms.

$$\begin{aligned}5(x+1) - 3(x+2) &= 5x+5-3x-6 \\ &= 2x-1\end{aligned}$$

$$\begin{aligned}(x+3)(x-4) &= x^2-4x+3x-12 \\ &= x^2-x-12\end{aligned}$$

Expand and simplify

1. $3(x+4) - 2(x+5)$

2. $2(x-1) - 3(x+2)$

3. $-5(x+4) - 2(x-3)$

4. $(2x-3)(x+1)$

5. $(3x-1)(x-2)$

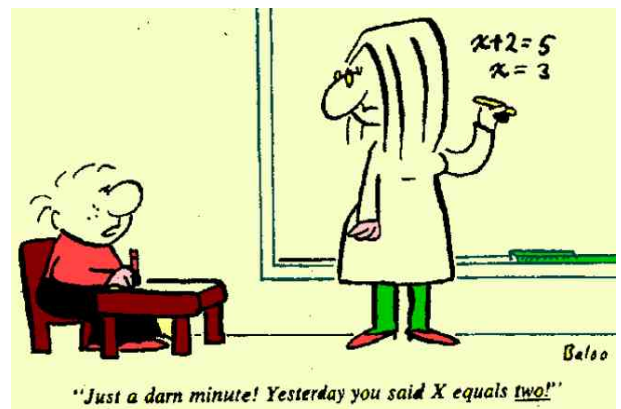
6. $(5x+1)(x-2)$

7. $(3x-1)(2-x)$

8. $3y^3 \times 5y^n = 15y^{12}$ What is the value of n ?

9. $4(y^2)^n \times 3y^4 = 12y^{16}$ Find the value of n .

10. $3b^2 \times 4b^n = 12b^8$ What is the value of n ?



Important Algebra Skills

6. Factorising

How to factorise

Take out the highest common factor from numbers.

Take out common letters with the highest power possible.

$$\begin{aligned} \text{Factorise } 9b^2 - 12bc &= 3(3b^2 - 4bc) \\ &= 3b(3b - 4c) \end{aligned}$$

If there are 3 terms the pattern might be quadratic.

$$\begin{aligned} \text{Factorise } x^2 + 9x - 36 &= (x \pm ?)(x \pm ?) \end{aligned}$$

Find a pair of numbers that multiply to make the end constant. (-36)

The pair of numbers must also add to make +9.

$$= (x - 3)(x + 12)$$

Factorise completely

1. $3c^2 + 15cd$

2. $2p^2 - 12pq$

3. $6a^3 - 8ax$

4. $x^2 - 2x - 8$

5. $x^2 - 5x - 14$

6. $x^2 + 7x - 60$

7. $x^2 + 7x + 10$

8. $x^2 + 7x + 12$

9. $p^2 + 2p - 15$

10. One factor of $x^2 + 48x - 100$ is $(x - 2)$ what is the other factor?

$$\sqrt{\heartsuit} = ? \quad \cos \heartsuit = ?$$

$$\frac{d}{dx} \heartsuit = ? \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \heartsuit = ?$$

$$F\{\heartsuit\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{it\heartsuit} dt = ?$$

My normal approach
is useless here.

Important Algebra Skills

7. Solving factorised equations

How to solve factorised equations

If we can factorise an equation to the form $A \cdot B = 0$, then either $A = 0$ or $B = 0$ or possibly both.

Solve $5x(x - 4) = 0$

In this case “A” is $5x$ and “B” is $x - 4$.

Either $5x = 0$ giving $x = 0$

Or $x - 4 = 0$ giving $x = 4$

Solve $(1 - 2x)(x + 3) = 0$

Either $1 - 2x = 0$ Or $x + 3 = 0$

$1 = 2x$

$x = -3$

$x = \frac{1}{2}$

Solve

1. $3x(x + 4) = 0$

2. $6x(x - 4) = 0$

3. $5m(m - 3) = 0$

4. $5y(8 - 2y) = 0$

5. $(x + 1)(x - 7) = 0$

6. $(x + 1)(4x - 1) = 0$

7. $(2x - 1)(x + 3) = 0$

8. $(1 - 2x)(x + 3) = 0$

9. $(1 - 4x)(x + 5) = 0$

10. $(3 - 6b)(2b - 8) = 0$



“Algebra class will be important to you later in life because there’s going to be a test six weeks from now.”

Important Algebra Skills

8. Fraction simplification

How to simplify algebraic fractions

In numbers, cancel highest common factors.

$$\frac{12}{18} = \frac{\mathbf{6}x\mathbf{2}}{\mathbf{6}x\mathbf{3}} = \frac{2}{3}$$

In variables, subtract powers.

$$\frac{12x^5}{18x^2} = \frac{2x^3}{3}$$

Factorise top and bottom before cancelling.

$$\frac{x^2 + 5x - 14}{3x - 6} = \frac{(x + 7)(x - 2)}{3(x - 2)} = \frac{x + 7}{3}$$

Simplify

1. $\frac{9x^5}{12x^3}$

2. $\frac{18x^7}{6x^3}$

3. $\frac{4m^5}{12m^2}$

4. $\frac{4m^5n}{12m^2}$

5. $\frac{15ab^5}{25a^3b^7}$

6. $\frac{x^2 + 7x + 10}{x + 2}$

7. $\frac{x^2 + 7x + 12}{x + 3}$

8. $\frac{p^2 + 2p - 15}{p - 3}$

9. $\frac{2p^2 - 12pq}{6p^2}$

10. $\frac{6a^3 - 8ax}{18a^3}$



Important Algebra Skills

9. Fraction operations

How to add, multiply, or divide algebraic fractions

Adding/subtracting—get a common denominator.

$$\frac{2x}{3} + \frac{x}{5} = \frac{5x2x}{5x3} + \frac{3x}{3x5} = \frac{10x + 3x}{15} = \frac{13x}{15}$$

Multiplying—multiply along the top and along the bottom.

$$\frac{2x}{3} \cdot \frac{x}{5} = \frac{2x^2}{15}$$

Dividing—multiply by the reciprocal.

$$\frac{2x}{3} \div \frac{x}{5} = \frac{2x}{3} \times \frac{5}{x} = \frac{10x}{3x} = \frac{10}{3} = 3\frac{1}{3}$$

Simplify

1. $\frac{2m}{3} + \frac{4m}{5}$

2. $\frac{x}{3} + \frac{x}{5}$

3. $\frac{3m}{5} - \frac{4m}{7}$

4. $\frac{x}{3} + \frac{4x}{9}$

5. $\frac{a}{2} + \frac{3}{b}$

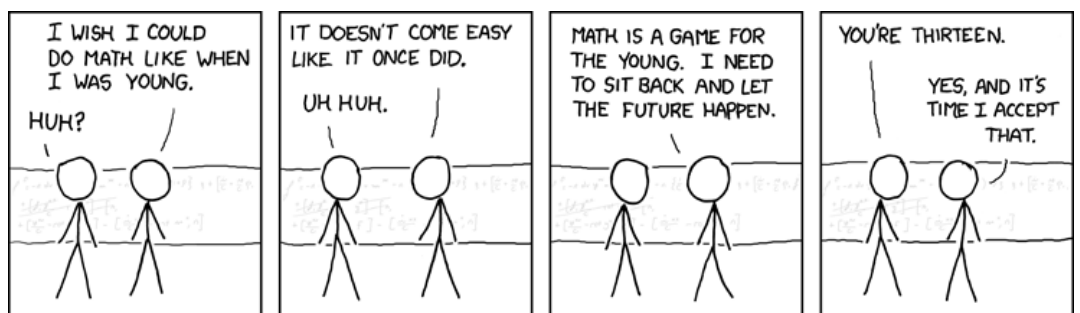
6. $\frac{12}{x} - \frac{2}{x}$

7. $\frac{12}{x} \cdot \frac{2}{x}$

8. $\frac{3}{4} \cdot \frac{6}{x^2}$

9. $\frac{4}{3} \cdot \frac{x+1}{x-2}$

10. $\frac{2x}{5} \div \frac{x}{2}$



Answers- Important Algebra Skills

1. Using a formula (substitution)

1. 176 2. 21 3. 2.68203×10^{16} 4. 435 5. 33.5 cm^3 (1dp)
6. 5.7 m (1dp) 7. 510 064 471 km^2 8. 2.84 s (2dp)

2. Linear equations with brackets

1. $x = 7$ 2. $t = -2\frac{1}{4}$ (-2.25) 3. $x = 1\frac{1}{2}$ (1.5) 4. $y = 8\frac{1}{5}$ (8.2) 5. $t = -\frac{1}{5}$ (-0.2)
6. $n = -5$ 7. $n = -6$ 8. $n = 4\frac{2}{5}$ (4.4) 9. $x = 7$ 10. $x = -1\frac{3}{5}$ (-1.6)

3. Linear equations with fractions

1. $x = 3\frac{1}{5}$ (3.2) 2. $w = 27$ 3. $x = 7\frac{1}{4}$ (7.25) 4. $x = 9\frac{1}{3}$ (9.33) 5. $x = -3\frac{1}{5}$ (-3.2)
6. $m = 35$ 7. $x = 3\frac{4}{5}$ (3.8) 8. $x = -22\frac{3}{4}$ (-22.75) 9. $x = 12$ 10. $w = 13\frac{4}{5}$ (13.8)

4. Linear equations collecting like terms

1. $x = 1\frac{3}{5}$ (1.6) 2. $x = -2\frac{1}{4}$ (-2.25) 3. $m = 4$ 4. $m = -\frac{1}{4}$ (-0.25) 5. $x = 4\frac{1}{2}$ (4.5)
6. $x = -3$ 7. $m = 10$ 8. $n = 1\frac{2}{3}$ (1.66) 9. $m = 2.2$ 10. $a = -1$

5. Multiplying and expanding

1. $x + 2$ 2. $-x - 8$ 3. $-7x - 14$ 4. $2x^2 - x - 3$ 5. $3x^2 - 7x + 2$
6. $5x^2 - 9x - 2$ 7. $-3x^2 + 7x - 2$ 8. 9 9. 6 10. 6

6. Factorising

1. $3c(c + 5d)$ 2. $2p(p - 6q)$ 3. $2a(3a^2 - 4x)$ 4. $(x + 2)(x - 4)$ 5. $(x + 2)(x - 7)$
6. $(x - 5)(x + 12)$ 7. $(x + 2)(x + 5)$ 8. $(x + 3)(x + 4)$ 9. $(p - 3)(p + 5)$ 10. $(x + 50)$

7. Solving factorised equations

1. $x = 0$ $x = -4$ 2. $x = 0$ $x = 4$ 3. $m = 0$ $m = 3$ 4. $y = 0$ $y = 4$ 5. $x = -1$ $x = 7$
6. $x = -1$ $x = \frac{1}{4}$ 7. $x = \frac{1}{2}$ $x = -3$ 8. $x = \frac{1}{2}$ $x = -3$ 9. $x = -5$ $x = \frac{1}{4}$ 10. $b = \frac{1}{2}$ $b = 4$

8. Fraction simplification

1. $\frac{3x^2}{4}$ 2. $3x^4$ 3. $\frac{m^3}{3}$ 4. $\frac{m^3n}{3}$ 5. $\frac{3}{5a^2b^2}$
6. $x + 5$ 7. $x + 4$ 8. $p + 5$ 9. $\frac{p - 6q}{3p}$ 10. $\frac{3a^2 - 4x}{9a^2}$

9. Fraction operations

1. $\frac{6m}{5}$ 2. $\frac{8x}{15}$ 3. $\frac{m}{35}$ 4. $\frac{7x}{9}$ 5. $\frac{ab + 6}{2b}$
6. $\frac{10}{x}$ 7. $\frac{24}{x^2}$ 8. $\frac{9}{2x^2}$ 9. $\frac{4x + 4}{3x - 6}$ 10. $\frac{4}{5}$